A New Algorithm to Minimize Nonlinear Effects in RF Traps

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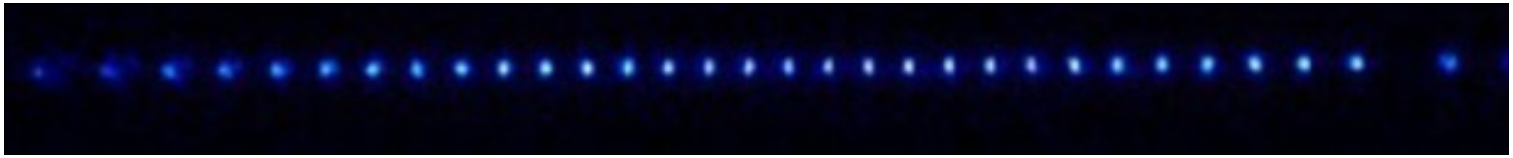
The Field of Atom Trapping

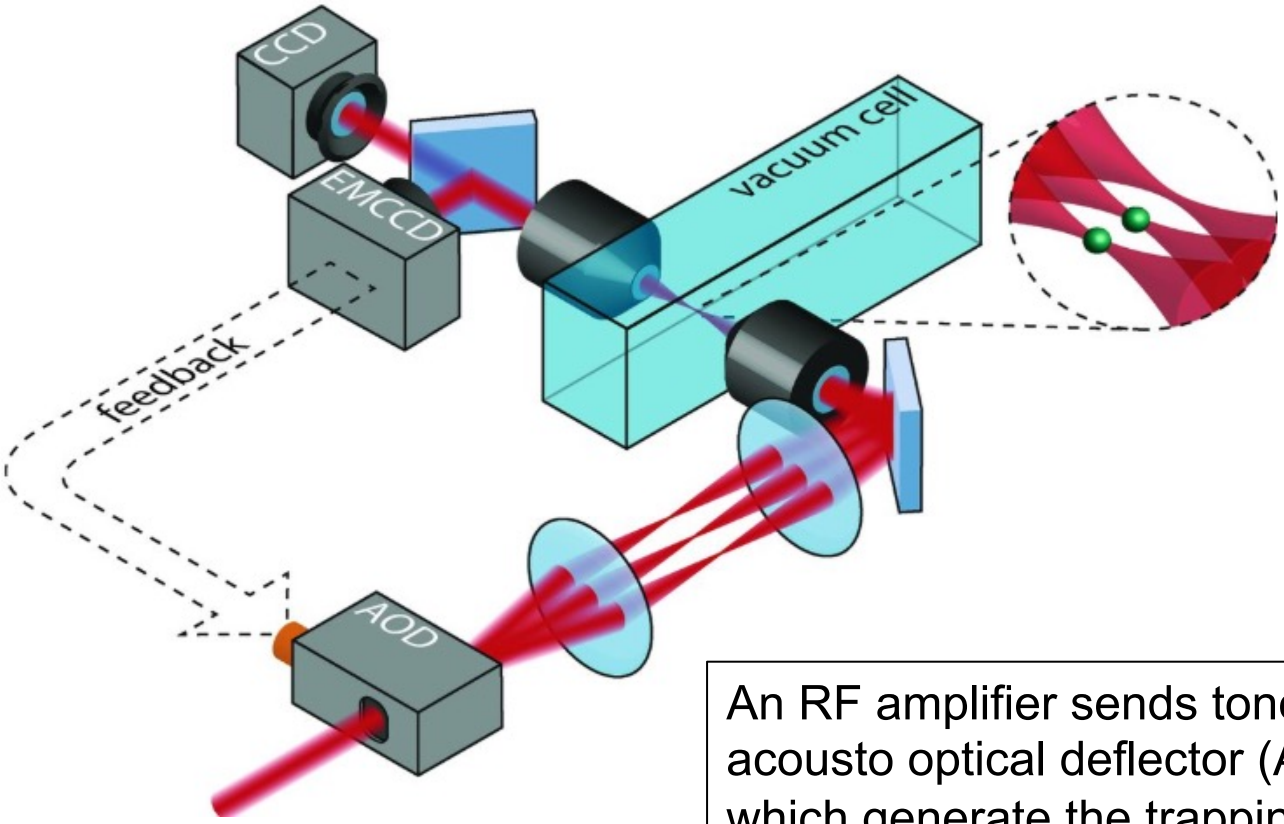
Trapping atoms and organizing them into lattices is critical for many fields:

- Quantum information and quantum computing
- Quantum many body theory



Source: https://sites.lsa.umich.edu/kuzmich-lab/

The Principles of Atom Trapping



An RF amplifier sends tones to an acousto optical deflector (AOD), which generate the trapping beams. How are the RF tones controlled and generated?

Source: DOI: 10.1126/science.aah3752

Generating Trapping Frequencies

- An RF amplifier generates N evenly spaced tones, where N is the number of traps.

$$E = \sum_{i=1}^{N} A_i e^{i(\omega_i t + \phi_i)}$$

- Two tones mix to generate to form a difference tone.

$$A_i e^{i(\omega_i t + \phi_i)}$$

$$A_j e^{i(\omega_j t + \phi_j)}$$

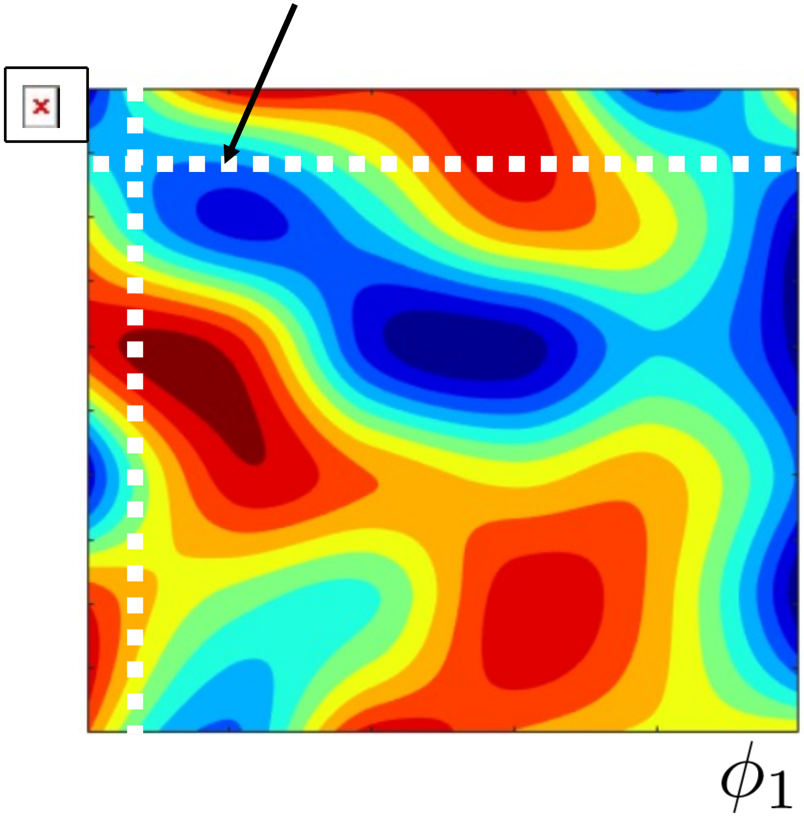
$$A_{ij} e^{i((\omega_i - \omega_j)t + (\phi_i - \phi_j))}$$

- This difference tone can then mix with an original tone to produce an unwanted tone.

The Task

- We must tune each phase to minimize the strength of the first order nonlinear effects.
- Original method: "We then optimize each phase, one at a time…"
  - Not rigorous or guaranteed to yield accurate results
- I will devise a new algorithm guaranteed to minimize the nonlinear effects.
  - Must be effective and computationally efficient
    - Competing with an O(N) algorithm.

Concludes the minimum is here



Methodology Through an Example

$$\omega_0 = 1$$

$$\Delta\omega = 1$$

$$\phi_i = random$$

$$A_i = 1$$

$$E_0 = \sin(t + \phi_1) + \sin(2t + \phi_2) + \sin(3t + \phi_3)$$

Nonlinear Effects in Matrix Form

$$
\begin{array}{c c}
 & \begin{array}{ccc} \sin(t+\phi_1) & \sin(2t+\phi_2) & \sin(3t+\phi_3) \end{array} \\
\begin{array}{c} \sin(t+\phi_1) \\ \sin(2t+\phi_2) \\ \sin(3t+\phi_3) \end{array} &
\begin{bmatrix} 0 & A_{12}\sin(t+(\phi_1-\phi_2)) & A_{13}\sin(2t+(\phi_1-\phi_3)) \\ 0 & 0 & A_{23}\sin(t+(\phi_2-\phi_3)) \\ 0 & 0 & 0 \end{bmatrix}
\end{array}
$$

Sum of all first order nonlinear effects:

$$E = A_{12}\sin(t+(\phi_1-\phi_2)) + A_{13}\sin(2t+(\phi_1-\phi_3)) + A_{23}\sin(t+(\phi_2-\phi_3))$$

This is a periodic signal with
$T=2\pi/\Delta\omega= 2\pi$

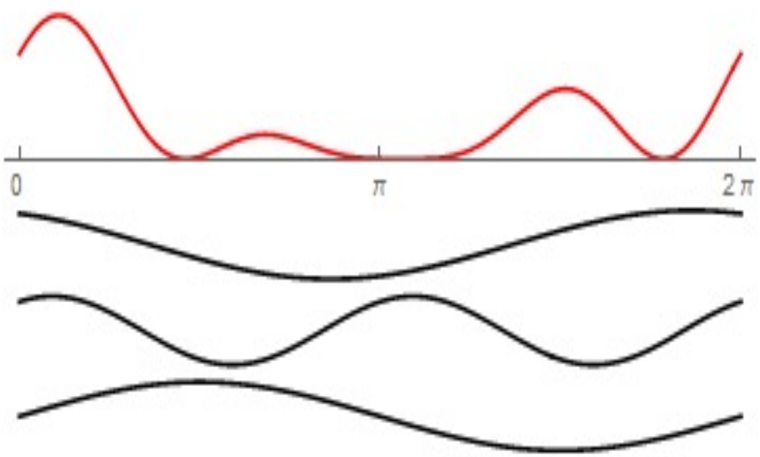
The Cost Function

$$(\sum_{i,j} A_{ij} \sin((\omega_i - \omega_j)t + (\phi_i - \phi_j)))^2$$

We introduce the cost function

$$\int_0^{\frac{2\pi}{\Delta\omega}} (\sum_{i,j} A_{ij} \sin((\omega_i - \omega_j)t + (\phi_i - \phi_j)))^2 dt$$

Which is the squared integral of the nonlinear effects, corresponding to the energy of the difference tone signal.



$$A_{12} \sin(t + (\phi_1 - \phi_2))$$

$$A_{13} \sin(2t + (\phi_1 - \phi_3))$$

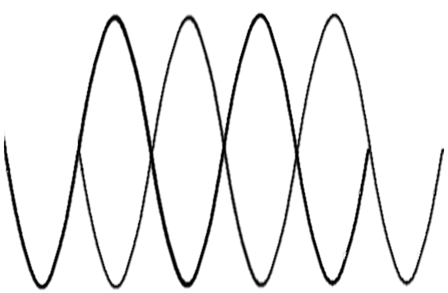
$$A_{23} \sin(t + (\phi_2 - \phi_3))$$

Minimizing the Cost Function

$$\int_0^T \sin(mx)\sin(nx) = 0$$

$$\int_0^T \sin(mx+\phi_1)\sin(nx+\phi_2) = ?$$

$$n \neq m$$



Audience Quiz: what does the integral equal now?

 = zero

 = non-zero

We can harness orthogonality to turn the integral of

$$\left(\sum_{i,j} A_{ij}\sin((\omega_i-\omega_j)t+(\phi_i-\phi_j))\right)^2$$

into a discrete sum.

If two terms of the same frequency are multiplied and integrated from 0 to $2\pi/\Delta\omega$, one obtains:

$$A_{ij}A_{kl}\frac{\pi}{\Delta\omega}\cos(\phi_i-\phi_j-(\phi_k-\phi_l))$$

Minimizing the cost function (cont.)

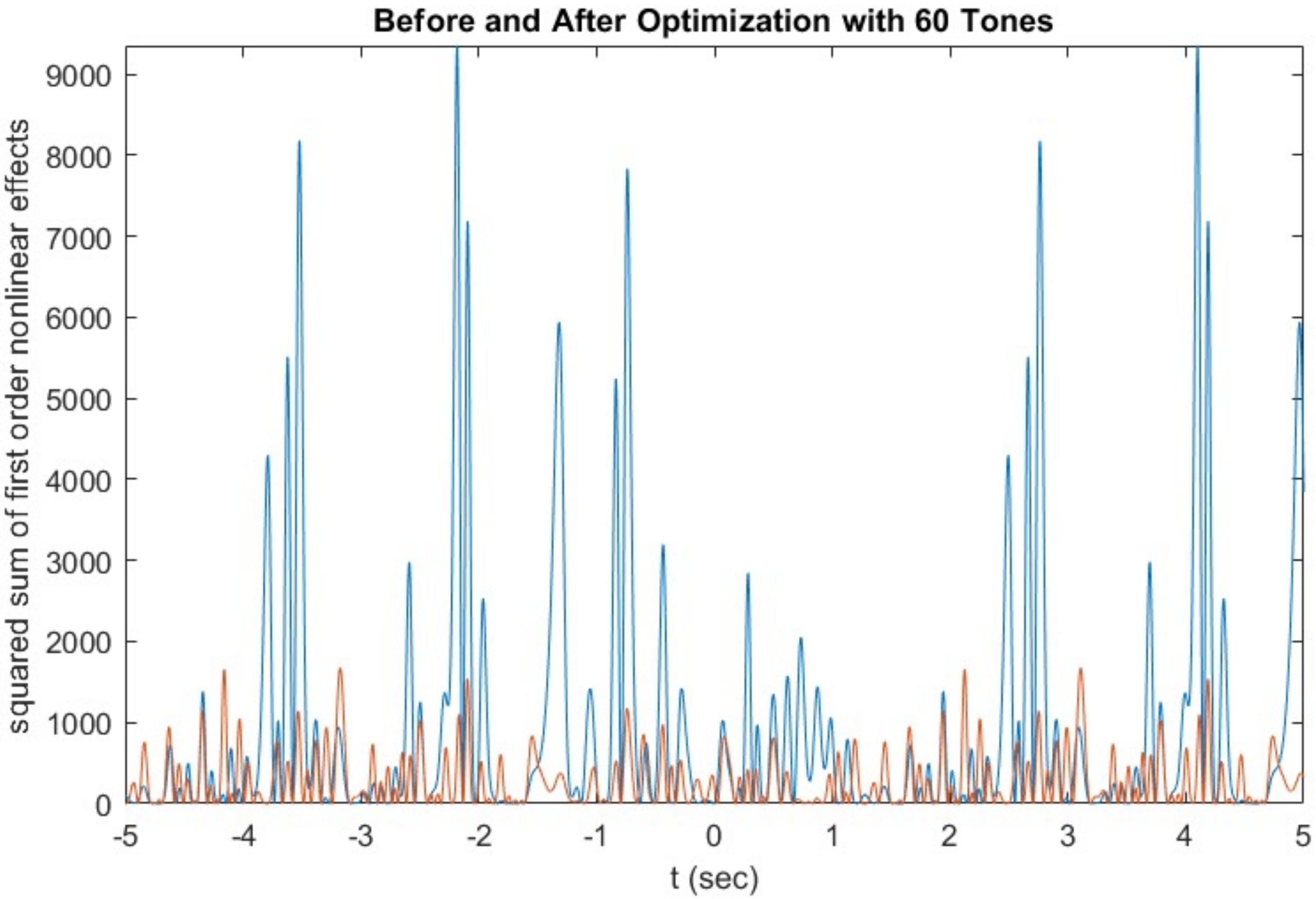
- Thus, to obtain the cost, one simply needs to sum all the

$$A_{ij} A_{kl} \frac{\pi}{\Delta\omega} \cos(\phi_i - \phi_j - (\phi_k - \phi_l))$$

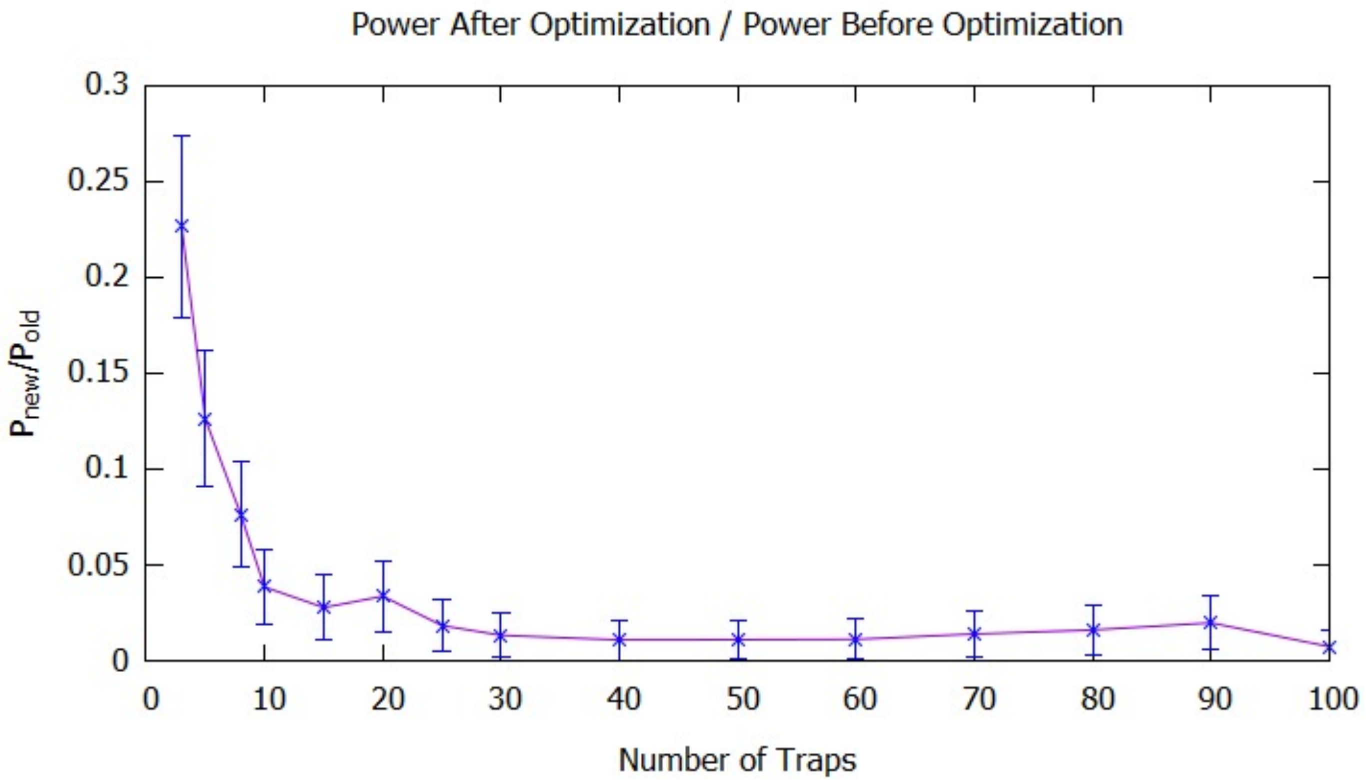
For all i,j,k,l such that $\omega_i - \omega_j = \omega_k - \omega_l$.

- We now have transformed an integration into a discrete sum. Additionally, explicitly calculating the gradient is now possible, allowing us to proceed using gradient descent algorithms.

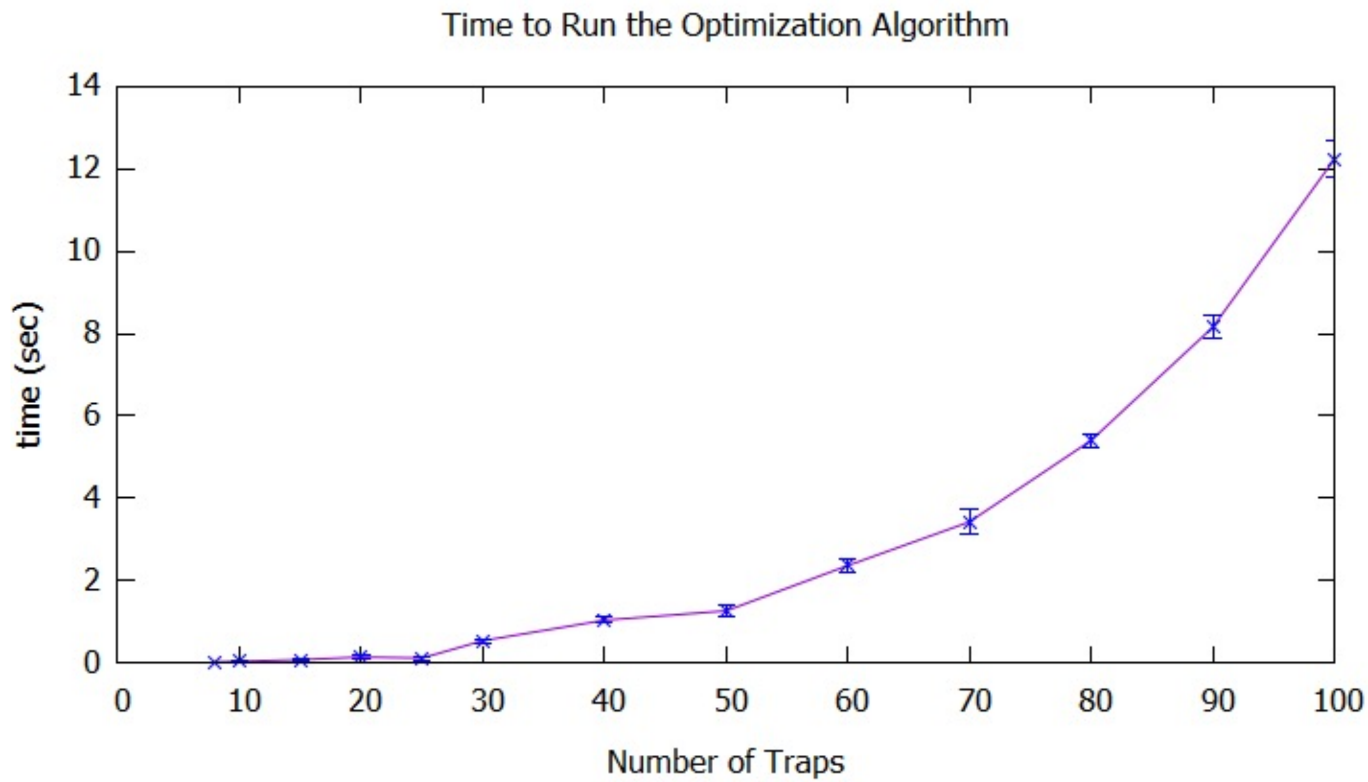
Results


Before and After Optimization with 60 Tones
squared sum of first order nonlinear effects
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
t (sec)

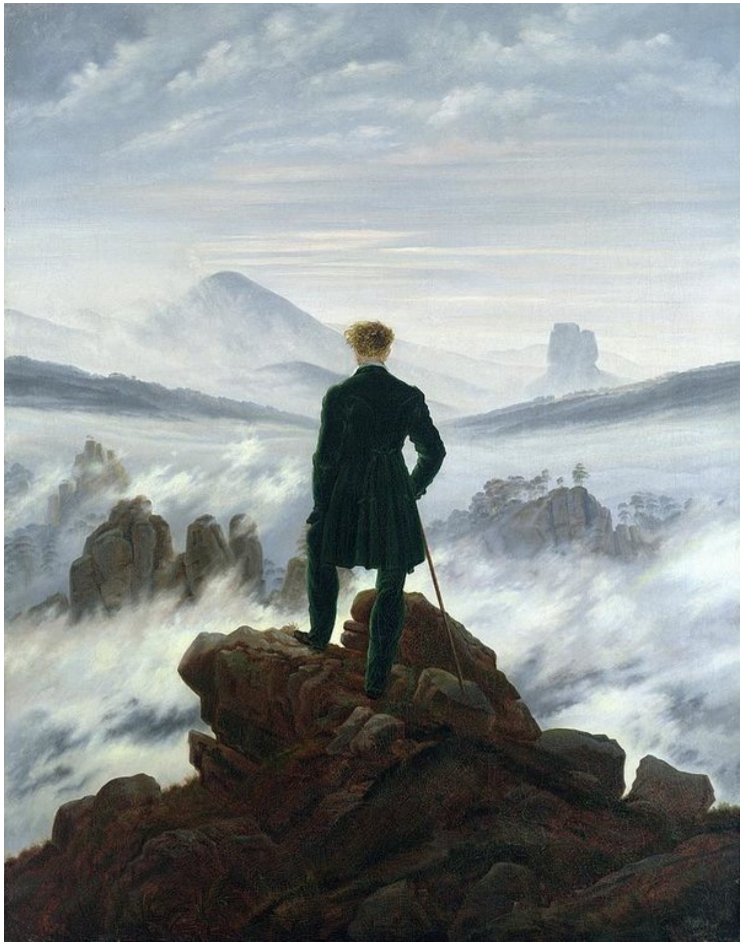
Quantifying Performance



Quantifying Performance



Closing Remarks



- I am humbled by my research, and the people that made this work possible.
  - The REU coordinators: Dr. Campbell, Dr. Liu, Dr. Zochowski.
  - My mentor, Dr. Kuzmich, and the lab members I worked closely with (Huy, Hikaru, Andrea, Brian).
  - My fellow REU students.

Source: https://fineartamerica.com/featured/the-wanderer-above-the-sea-of-fog-caspar-david-friedrich.html?product=poster