University of Michigan REU Report: A New Algorithm to Minimize Nonlinear Effects in RF Traps

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1 Introduction

Over the summer, I worked with the Kuzmich group to improve the software required for atom trapping. My research experience can be split roughly into two phases: in the first half of the program, I acclimated myself to the field by studying keystone papers and supplementary material provided by lab members. In the latter half of the program, I solved a complex computational optimization problem, the solution of which can be implemented in the RF (radio frequency) amplifiers once the physical experiment resumes.

2 First Phase

I began my REU by meeting the lab members I would work closely with throughout the summer: Huy Nguyen and Hikaru Tamura. Both have an intimate understanding of atom trapping and guided me through my attempt to get a foothold in this field. A foundational technique in this discipline is creating high fidelity optical traps that are capable of levitating atoms in a closely packed, highly organized lattice. While I had some experience in optical trapping, it was primarily related to manipulating a single macroscopic object. The hardware and optics required to levitate multiple atoms was foreign to me, so I went through a 2-3-week adjustment period where I studied three keystone papers [1,2,3] and presented them to my lab group. References 1 and 2 were critical to my understanding but ended up not being relevant to my research project. Reference 3 provides excellent exposition to my project and a nice summary of what I learned during this phase, so I will summarize its most important points below.

Referring to figure 1, a laser beam is first sent through an acousto-optical deflector (AOD). This device creates standing sound waves; in other words, the laser enters a medium with a dynamically adjusted index of refraction. The AOD generates an arbitrary number of beams that are shifted in frequency.

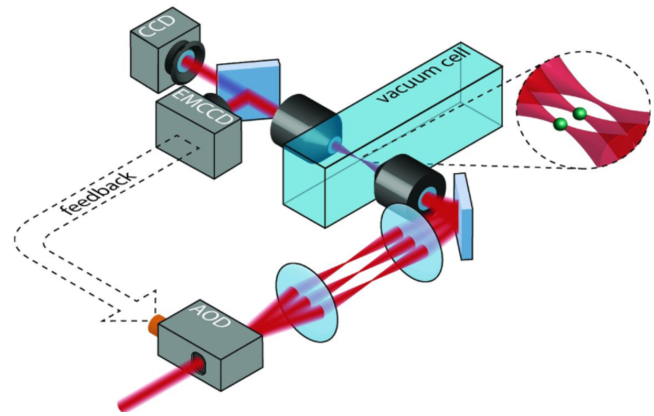

Figure 1: Figure 1: Experimental setup for [3], illustrating the most critical components and their relation to one another.

These beams are then sent through a simple telescope to change their size and are subsequently channeled through trap-shaping optics. The beams form evenly spaced microscopic traps in a vacuum cell. Within the vacuum cell are gaseous rubidium atoms that have been laser cooled (the atoms must be cold enough as to not escape the traps due to their thermal kinetic energy). The atoms are then stochastically loaded into these traps. The trapped rubidium atoms fluoresce, which is observed by the CCD cameras. Thus, the state of the system can be captured by the CCDs and coupled to the AOD, allowing for the traps to be adjusted based on their occupancy. Not shown in this figure is an RF amplifier, which supplies the AOD with sinusoidal electrical tones. These tones drive the AOD to create multiple trapping beams from a single beam. The next phase of my research project involved studying the RF amplifier, and how it can generate noise-free tones that result in the creation of stable and uniform trapping beams.

3 Second Phase

Second Phase: To drive the AOD, and RF amplifier generates N evenly spaced electrical tones, where N is the number of trapping beams. This can be represented mathematically by

$$E = \sum_{i=1}^{N} A_i e^{i(\omega_i t + \phi_i)}. \tag{1}$$

Each constituent tone has its own amplitude, frequency, and phase. The amplitudes and phases can be arbitrarily chosen and generated. The frequencies are evenly spaced by $\Delta\omega$. Two arbitrary tones (say i and j) can mix to produce a *first order difference tone (FODT)*, represented below. This FODT can then mix

$$A_i e^{i(\omega_i t+\phi_i)}, \quad A_j e^{i(\omega_j t+\phi_j)} \longrightarrow A_{ij} e^{i((\omega_i-\omega_j)t+(\phi_i-\phi_j))}$$

Figure 2: Two frequencies mixing to generate a first-order difference tone

with the original tones to produce unwanted effects. For example, consider when the FODT mixes with the original tone i. These two tones can produce a second order difference tone, which has a frequency of ω_j. This tone can destructively interfere with the original tone of frequency ω_j, meaning the RF amplifier sends a weaker tone to the AOD. The trapping beam corresponding to the frequency ω_j would then no longer be strong enough to trap atoms. Moreover, thre traps would no longer be uniform in depth. Thus, it is imperative to minimize the FODTs to reduce their capability of producing unwanted interactions.

I referred to the supplementary material in [3] to study and potentially emulate their methods of reducing the FODTs. Their method involved tuning the phases of the original tones to minimize the sum of the FODTs. Put in another way, by fine tuning the phases, one can make the FODTs partially destructively interfere. The paper described how they "optimize[d] each phase, one at a time..." to accomplish this task. This is not a rigorous way to solve this problem, so it does not guarantee accurate results. Additionally, the efficacy of this method is reduced as the number of traps is increased, since the sum of the FODTs becomes more complex. I had the idea of using a gradient descent algorithm to minimize the sum of the FODTs. At the cost of computational efficiency, my method is guaranteed to converge to a local minimum. The original method is O(N), since each phase is considered separately. My method is necessarily slower than an O(N) algorithm. However, this will not be an issue as long as my method runs in a reasonable amount of time for the number of tones that we are concerned with (around 40-100).

My methodology is best understood through an example. Consider generating three tones with an RF amplifier. The principal frequency is 1 rad/sec, the frequency spacing ($\Delta\omega$) is 1 rad/sec, the phases are initialized randomly, and each amplitude is 1. The RF amplifier creates a signal

$$E = \sin(t+\phi_1) + \sin(2t+\phi_2) + \sin(3t+\phi_3). \tag{2}$$

These three tones mix with one another to produce three difference tones, which we can represent using a matrix:

$$\begin{array}{c|ccc} & \sin(t+\phi_1) & \sin(2t+\phi_2) & \sin(3t+\phi_3) \\ \hline \sin(t+\phi_1) & 0 & A_{12}\sin(t+(\phi_1-\phi_2)) & A_{13}\sin(2t+(\phi_1-\phi_3)) \\ \sin(2t+\phi_2) & 0 & 0 & A_{23}\sin(t+(\phi_2-\phi_3)) \\ \sin(3t+\phi_3) & 0 & 0 & 0 \end{array}$$

Figure 3: Nonlinear effects in matrix form.

The matrix contains zeros on the diagonal, since a tone cannot mix with itself. Additionally, to avoid double counting, we set the lower triangular elements of the matrix equal to zero. One can calculate the sum of all the FODTs by adding each element in this matrix. Note that the sum of the FODTs is periodic- two original tones that are spaced $\Delta\omega$ apart mix to produce a FODT with the smallest possible frequency, namely $\Delta\omega$. All other combinations of original tones produce a FODT that is an integer multiple of $\Delta\omega$. We see this in our example with three frequencies; the sum of each element in the matrix has a period of 2π.

With this in mind, we can now introduce a cost function f that we wish to minimize:

$$f = \int_0^{\frac{2\pi}{\Delta\omega}} (\sum_{i,j=1}^{N} A_{ij} \sin((\omega_i - \omega_j)t + (\phi_i - \phi_j)))^2. \tag{3}$$

Qualitatively, f corresponds the energy contained in one period of the total first order difference tone signal. In its current form, optimizing f is intractable since every iteration would require multiple numerical integrations. Fortunately, we can exploit orthogonality to transform f into a discrete sum. The integrand of f is a sum of sine waves multiplied by itself. We can expand f and only integrate terms of the form[1]

$$A_{ij}A_{kl} \sin((\omega_i - \omega_j)t + (\phi_i - \phi_j)) \sin((\omega_k - \omega_l)t + (\phi_k - \phi_l)) \mid \omega_i - \omega_j = \omega_k - \omega_l \tag{4}$$

since all other terms are orthogonal and integrate to zero. After integrating and simplifying algebraically, we obtain a much more computationally tractable cost function:

$$f = \sum_{i,j,k,l=1}^{N} A_{ik}A_{kl} \frac{\pi}{\Delta\omega} \cos(\phi_i - \phi_j - \phi_k + \phi_l) \mid \omega_i - \omega_j = \omega_k - \omega_l. \tag{5}$$

Additionally, this function contains the phases as arguments of elementary functions, meaning we can also calculate its gradient explicitly. By supplying the gradient, we significantly improve computation time and can make use of more sophisticated gradient descent algorithms. My code was written in MATLAB, where I made use of the *trust-region* minimization tool to run my program [4].

Figure 4 demonstrates the efficacy of my final algorithm. In less than 3 seconds, it optimized the phases to transform the original (blue) signal to the minimized (orange) signal. As shown in the plot, the major spikes in the original signal are eliminated entirely, meaning the main contributions to the first order nonlinear effects have been significantly reduced.

[1] "|" denotes "such that"

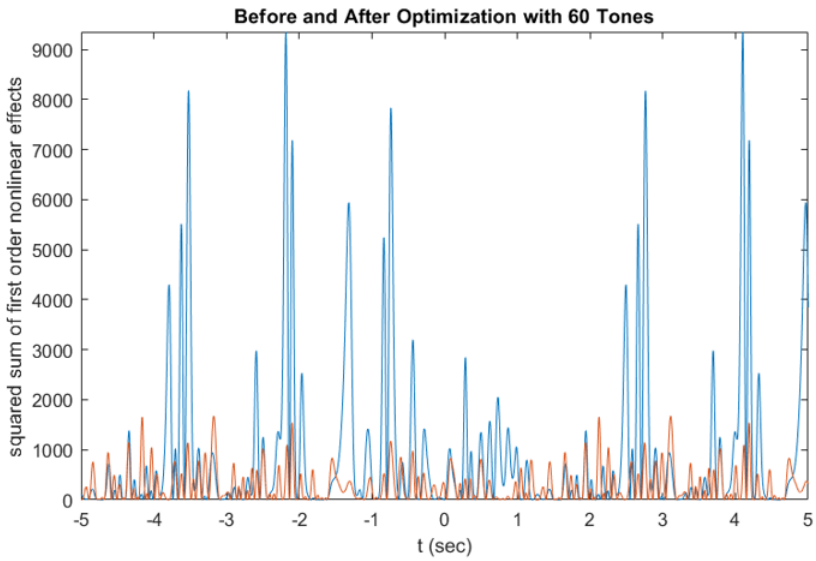

Figure 4: My algorithm is capable of significantly reducing integral of the signal over a period. The blue curve represents the signal without phase optimization, and the orange curve represents the signal after phase optimization.

Lastly, figure 5 shows the computational efficiency of my algorithm as a function of the number of traps. As evidenced by the clear upwards curvature in the data points, my algorithm is less efficient than an O(N) algorithm.[2] Nevertheless, for the number of traps that the Kuzmich group is concerned with (40-100), the program runs in a reasonable amount of time. Whether this time is sufficient is still an open question; we will know once the physical experiment is revived.

[2]The growth pattern of this function doesn't match a polynomial or an exponential, so it is difficult to characterize with big O notation.

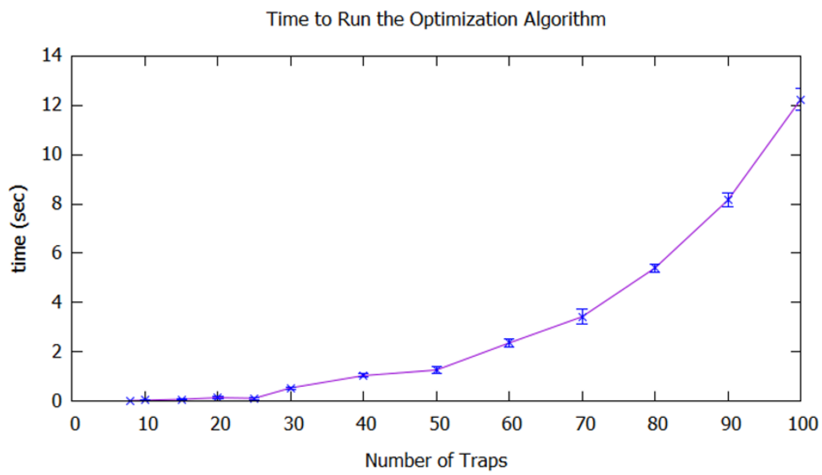

Figure 5: My algorithm is less efficient than an O(N) algorithm, but nevertheless runs in a reasonable amount of time.

4 Conclusion

As part of the University of Michigan's REU program, I was introduced to the field of atom trapping and devised a new algorithm to minimize the nonlinear effects that arise in RF traps. I am incredibly humbled to have been given this opportunity, and I hope that was able to demonstrate my gratitude through my hard work. This has been one of the most rewarding experiences of my undergraduate career, and it solidified my commitment to becoming a fully realized physicist. I was able to improve on skills that I was sorely lacking, such as programming, algorithmic thinking, and working independently. I would like to sincerely thank Dr. Campbell and Dr. Liu for organizing and running this program. They handled the online transition with grace, and made this experience constructive and worthwhile despite the current circumstances. My mentor, Dr. Kuzmich, and his lab members, Huy and Hikaru, also deserve many thanks, as they were supportive of me through the entire program. As disappointing as it was to not be on Michigan's campus this summer, I hope to be there next spring when I visit it as a prospective graduate student.

5 References

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