Broken Rotational Invariance in AdS/CFT

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What is AdS/CFT?

• **AdS** = “Anti de Sitter”
  – A maximally symmetric solution to Einstein’s field equations in vacuum

• **CFT** = “Conformal Field Theory”
  – Quantum Field Theory that is symmetric under conformal transformations

• **AdS/CFT correspondence**
  – $d$-dimensional CFTs have dual descriptions in an $d+1$-dimensional AdS space
  – CFT lives on the boundary of the AdS space (holographic)
Why is AdS/CFT interesting to us?

• Strongly coupled field theories are challenging
• Strongly-coupled field theory = Weak curvature gravitational theory
• Use techniques from GR to study field theories
• AdS/CMT
AdS/CMT

- Are condensed matter systems appropriately described by a conformal field theory?

\[ i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \right] \Psi(x, t) \]

- No
  - Dilation symmetry broken
  - Lorentz symmetry broken
AdS/CMT

• Can we deform AdS/CFT for condensed matter applications?
  – Lifshitz Geometry

• Is this good enough?
  – We still have rotational symmetry in spatial dimensions
AdS/CMT

• Spatial rotational invariance not a general property of condensed matter systems
• We need to break this symmetry in our geometry also
• Brings us to our geometry of interest
  – Homogeneous in $t, x, y$
  – Anisotropic in $t, x, y$
  – Scaling/Bulk dimension given by $r$

$$ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dx^2}{r^{2p}} + \frac{dy^2}{r^{2q}} + \frac{dr^2}{r^2}$$
Energy Conditions

- Used to restrict to physical solutions of Einstein’s Field Equations
  - Mass-Energy density is nonnegative
- Field Equations relate matter distribution to geometry, so we can use the Field Equations and the energy conditions to constraint $z, p, q$
- INSERT CONSTRAINTS HERE

\[
T^{\mu \nu} = \begin{bmatrix}
\rho & 0 & 0 & 0 \\
0 & p & 0 & 0 \\
0 & 0 & p & 0 \\
0 & 0 & 0 & p \\
\end{bmatrix}
\]

NEC: $\rho + p \geq 0$

WEC: $\rho \geq 0, \rho + p \geq 0$
Klein-Gordon Equation

• Use Lagrangian Density

\[ \mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} g^{\mu \nu} \partial_\mu \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right] \]

• Vary with respect to field

\[ \partial_\mu \left[ \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \right] - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \]

• Find equation of motion

\[ -\frac{1}{\sqrt{-g}} \partial_\mu \left( g^{\mu \nu} \sqrt{-g} \partial_\nu \phi \right) + m^2 \phi = 0 \]
Schrodinger Potential

• We find a 2\textsuperscript{nd} order ODE for our field
  \[ -\phi''(r) + \alpha(r)\phi'(r) + \beta(r)\phi(r) = 0 \]

• An appropriate field redefinition provides
  \[ -\phi''(r) + \left( \frac{\nu^2 + k_x r^2 p + k_y r^2 q - \omega^2 r^2 z}{r^2} \right) \phi(r) = 0 \]

• This looks like the Time-Independent Schrodinger Equation!
  – Can use techniques from Quantum Mechanics
Sample Schrödinger Potential

\[ V(r) = \frac{\nu^2 + k_x r^{2p} + k_y r^{2q} - \omega^2 r^{2z}}{r^2} \]
WKB and Green Functions Results

\[ \phi_{WKB}(r) = C_{\pm} \frac{1}{|E - V(r)|^{\frac{1}{4}}} e^{\pm i \int_{r_0}^{r} \sqrt{E - V(r')} dr'} \]

- WKB approximation allows us to understand behavior past the classical turning point
- Prescription from AdS/CFT enables us to find Green Function for field based on behavior at \( r = 0 \) and \( r = \infty \)
- **FIND GREEN FUNCTION RESULT**
Conclusion

• First steps in understanding spatially anisotropic AdS/CFT

• Several obvious next steps
  – Flows between metrics
  – Full numerical computation
  – How do we understand unusual behavior near origin?
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