Broken Rotational Invariance in AdS/CFT

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What is AdS/CFT?

- AdS = “Anti de Sitter”
  - A maximally symmetric solution to Einstein’s field equations in vacuum
- CFT = “Conformal Field Theory”
  - Quantum Field Theory that is symmetric under conformal transformations
- AdS/CFT correspondence
  - d-dimensional CFTs have dual descriptions in an d+1-dimensional AdS space
  - CFT lives on the boundary of the AdS space (holographic)

Why is AdS/CFT interesting to us?

- Strongly coupled field theories are challenging
- Strongly-coupled field theory = Weak curvature gravitational theory
- Use techniques from GR to study field theories
- AdS/CMT

AdS/CMT

- Are condensed matter systems appropriately described by a conformal field theory?

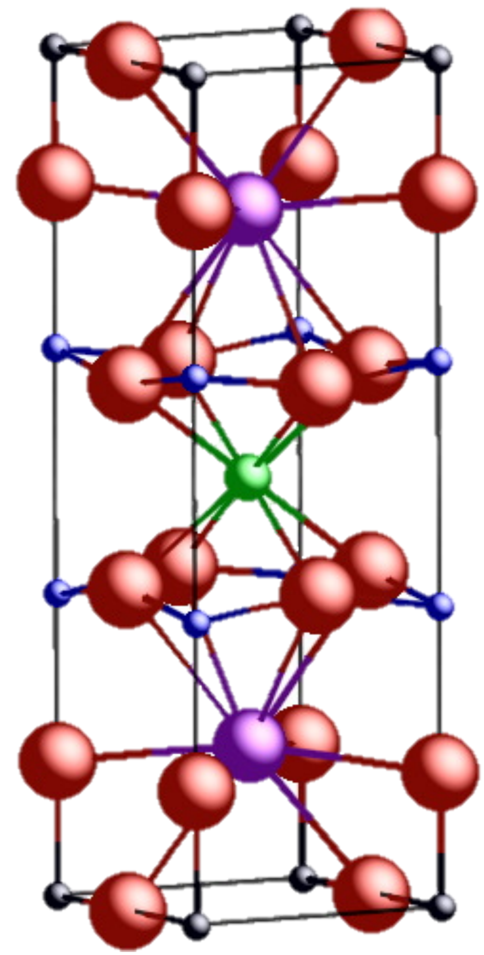
$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)\right]\Psi(x,t)$$

- No
  - Dilation symmetry broken
  - Lorentz symmetry broken

AdS/CMT

- Can we deform AdS/CFT for condensed matter applications?
  - Liftshitz Geometry

- Is this good enough?
  - We still have rotational symmetry in spatial dimensions



AdS/CMT

- Spatial rotational invariance not a general property of condensed matter systems
- We need to break this symmetry in our geometry also
- Brings us to our geometry of interest
  - Homogeneous in t, x, y
  - Anisotropic in t, x, y
  - Scaling/Bulk dimension given by r

$$ds^2 = \frac{-dt^2}{r^{2z}} + \frac{dx^2}{r^{2p}} + \frac{dy^2}{r^{2q}} + \frac{dr^2}{r^2}$$

Energy Conditions

- Used to restrict to physical solutions of Einstein's Field Equations
  - Mass-Energy density is nonnegative
- Field Equations relate matter distribution to geometry, so we can use the Field Equations and the energy conditions to constraint z, p, q
- INSERT CONSTRAINTS HERE

$$T^{\mu\nu} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix}$$

$$\text{NEC: } \rho + p \geq 0$$

$$\text{WEC: } \rho \geq 0,\ \rho + p \geq 0$$

Klein-Gordon Equation

- Use Lagrangian Density

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right]$$

- Vary with respect to field

$$\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial_\mu\phi}\right] - \frac{\partial\mathcal{L}}{\partial\phi} = 0$$

- Find equation of motion

$$\frac{-1}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu\phi\right) + m^2\phi = 0$$

Schrodinger Potential

- We find a 2nd order ODE for our field

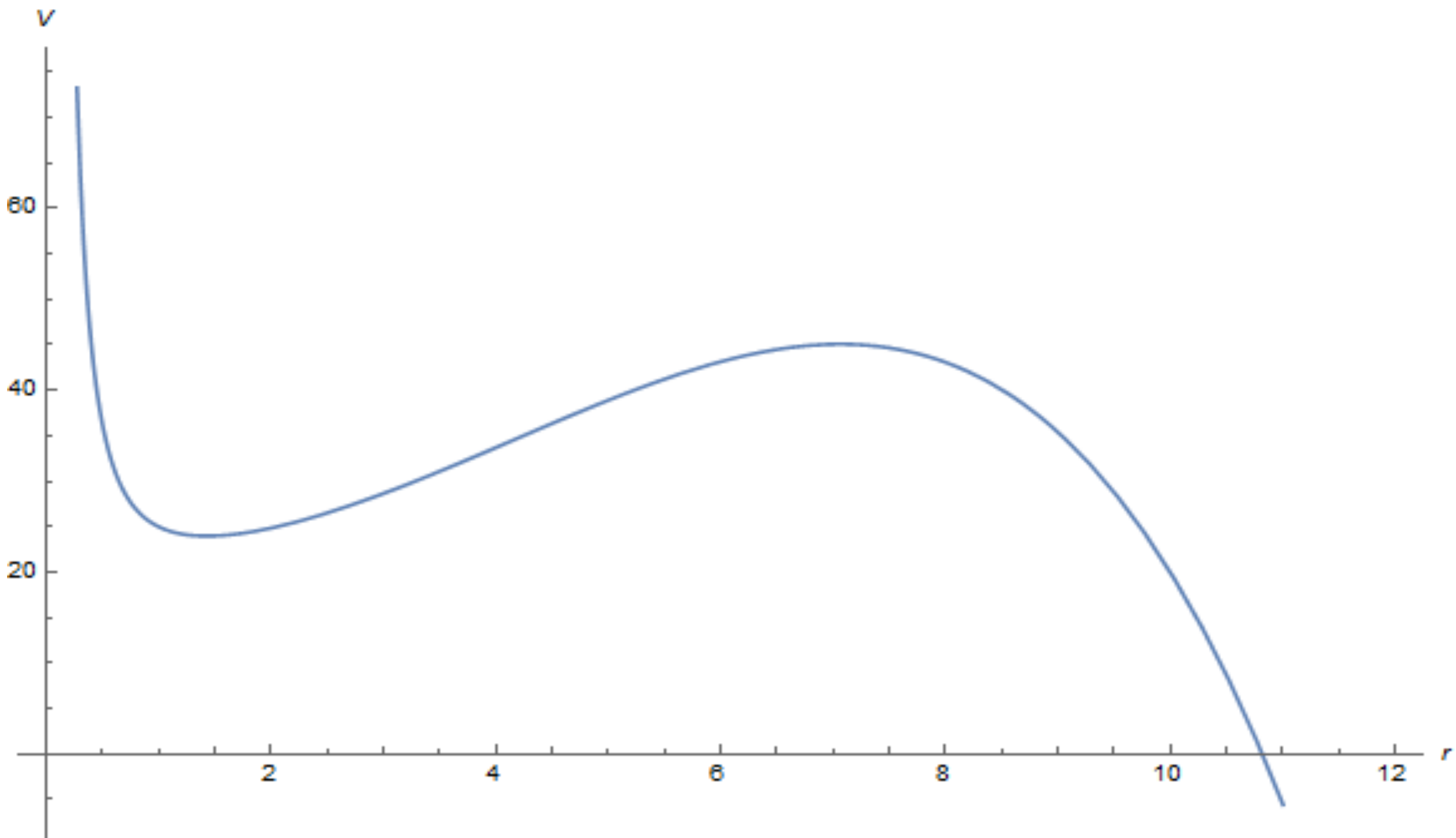
$$-\phi''(r) + \alpha(r)\phi'(r) + \beta(r)\phi(r) = 0$$

- An appropriate field redefinition provides

$$-\phi''(r) + \left(\frac{\nu^2 + k_x^2 r^{2p} + k_y r^{2q} - \omega^2 r^{2z}}{r^2}\right)\phi(r) = 0$$

- This looks like the Time-Independent Schrodinger Equation!
  - Can use techniques from Quantum Mechanics

Sample Schrodinger Potential



$$V(r) = \frac{\nu^2 + k_x^2 r^{2p} + k_y r^{2q} - \omega^2 r^{2z}}{r^2}$$

WKB and Green Functions Results

$$\phi_{WKB}(r) = C_{\pm} \frac{1}{|E - V(r)|^{\frac{1}{4}}} e^{\pm i \int_{r_0}^{r} \sqrt{E - V(r')} dr'}$$

- WKB approximation allows us to understand behavior past the classical turning point
- Prescription from AdS/CFT enables us to find Green Function for field based on behavior at at r = 0 and r = ∞
- FIND GREEN FUNTION RESULT

Conclusion

- First steps in understanding spatially anisotropic AdS/CFT
- Several obvious next steps
  - Flows between metrics
  - Full numerical computation
  - How do we understand unusual behavior near origin?

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