Wilson Loop in Matrix Model

Guojun Zhang

Advisors: Leopoldo A. Pando Zayas, James T. Liu

August 18, 2014

Abstract

In this paper, Wilson loops of fundamental representation, antisymmetric representation as well as symmetric case are studied. Using localization method, we can reduce a functional integral in $\mathcal{N} = 4 SU(N)$ supersymmetric Yang-Mills theory into an integration over matrices. Specially, we calculated the correction term of symmetric rank $k$ Wilson loop, which matches with the $D3$ brane calculation.

1 Introduction

The duality between SU(N) super Yang-Mills theory with $N \rightarrow \infty$ and the string theory has been a hot research topic since Juan Maldacena proposed his AdS/CFT conjecture. And the match between SUSYM Wilson Loop and fundamental string, or D-brane is one of the most important results of AdS/CFT. We did an exhaustive research on all kinds of Wilson loops in the Gaussian matrix model, including fundamental representation, symmetric and antisymmetric.

2 Correction Term in Symmetric Wilson Loop

Under the limit of $k \rightarrow \infty, N \rightarrow \infty, \frac{k}{N}$ fixed, we can calculate the Wilson loop in symmetric rank $k$ representation using the Hartnoll-Kumar’s prescription. First, with localization skill we have the partition function:

$$Z = \int dM \exp\left(-\frac{2N}{\lambda} Tr M^2\right) = \int \prod_i dm_i \Delta^2(m) e^{-2N \sum_{i=1}^N m_i^2}$$  (1)

where $\Delta(\lambda)$ is the Vandermonde determinant. Searching for the saddle point here we will find a Wigner semicircle distribution: $\rho(m) = \frac{2}{\pi \lambda} \sqrt{\lambda - m^2}(m \in [-\sqrt{\lambda}, \sqrt{\lambda}])$.

Also we define a generating function:

$$F(t) = \prod_i \frac{1}{1 - te^{m_i}}$$  (2)

And the Wilson loop for rank $k$ is just the coefficient of $t^k$ in the generating function. With residue theorem it is easy to pick out the coefficient, and after a change of variable the expression for symmetric Wilson loop with rank $k$ is:

$$\frac{\sqrt{\lambda}}{2\pi i} \int_C dz \exp(-N[\frac{2}{\pi} \int_{-1}^1 dx \sqrt{1 - x^2} \ln(1 - e^{\sqrt{\lambda}(-x^2)}) + f\sqrt{\lambda z}])$$  (3)
where the contour $C$ is shown as follows:

And this contour can be deformed as the sum of $C_{\text{branch}}$ and $C'$, but the integral over $C'$ goes to zero as $\text{Re} z \to \infty$, so,

$$
\int_C = \int_{C_{\text{branch}}} + \int_{C'}
$$

where $y_0$ is the saddle point, i.e., $g'(y_0) = 0$.

The final result is:

$$
\sqrt{\frac{8}{\pi f^2 N}} \exp\left(2N\left(\kappa \sqrt{1 + \kappa^2 + \sinh^{-1}\kappa}\right) + \frac{1}{2} \ln \left(\frac{\kappa^3}{\sqrt{1 + \kappa^2}}\right)\right)
$$

And the 1st, 2nd terms are exactly the same as the $D3$ brane calculation.

### 3 Other representations

We have also done some research on other representations, such as fundamental and antisymmetric representations. Here are some primary results:

1. Drukker and Gross have calculated the fundamental Wilson loop:

$$
\frac{1}{N} \exp\left(\frac{-\lambda}{8N}\right) L^1_{N-1} \sim \frac{\exp\left(\sqrt{\lambda}\right)}{\lambda^{3/4}}
$$

which matches with fundamental string calculation. If we make a change $\sqrt{\lambda} \to k \sqrt{\lambda}$, and expand the result we will get $\exp\left(2N\left(\kappa \sqrt{1 + \kappa^2 + \sinh^{-1}\kappa}\right) - \frac{1}{2} \ln (k^3 \sqrt{1 + \kappa^2})\right)$

$$(\kappa = \frac{k \sqrt{\lambda}}{4N})$$

which matches with the symmetric case to the leading order, but not to the next-to-leading order because there is some contribution from other terms.

2. In the realm of antisymmetric representation, Fiol and Torrents give us the exact answer, a sum over permutations of products of Laguerre polynomials, from which the $\exp\left(\sqrt{\lambda}\right)$ behavior is not obvious. Although our analytical approach from the exact result to the large $\lambda$ behavior is not very successful, some primary numerical work has shown that the exact result fits with the $D5$ brane calculation to the leading order. We have also demonstrated their is a relation between $SU(N)$ and $U(N)$

2
Wilson loop:

\[ W_{A_k}^{SU(N)} = \exp\left(-\frac{\lambda k^2}{8N^2}\right)W_{A_k}^{U(N)} \]  

\( \text{(7)} \)

Acknowledgement

The author would like to thank Professor James T. Liu and Leopoldo A. Pando Zayas for instruction and people in the UM high energy theory group for discussion and lectures. Thanks to USTC, UM Physics Department and NSF for necessary supports.

References


