Calculating Scattering Amplitudes and Form Factors

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Scattering

Scattering Amplitude: Probability of scattering particles as functions of angles and energy.
Quantum Field Theory and Feynman Rules

Quantum Field Theory is a mathematical framework for particle physics.

Quantum Chromodynamics – Theory of the strong interactions, describing the interactions between quarks and gluons.

Gluons can interact with each other!

Gluon Diagram

Gluon Scattering: $gg \rightarrow gg$

$gg \rightarrow ggg$

10 Diagrams

$gg \rightarrow gggg$

38 Diagrams

The number of diagrams increase!
Parke-Taylor Formula

Calculated amplitude for $gg \rightarrow gggg$ scattering (1985) and developed an amplitude for $n$ gluon scattering:

$$A_n[g_1^- g_2^- g_3^+ g_4^+ \cdots g_n^+] = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \cdots \langle n1 \rangle}$$

Each of the bracket expressions are compact forms of an expression that is a function of momentum and energy.

i.e. $\langle 12 \rangle \sim \sqrt{2E_1 E_2 - 2\vec{p}_1 \vec{p}_2} \ e^{i\phi}$
(Why?) BCFW Recursion Relations

- Britto, Cachazo, Feng, and Witten (2005)

- Recursion relations provide an efficient way of calculating higher point amplitudes.

- The diagrams are not Feynman diagrams, but serve the same purpose.
My Project

- To apply BCFW recursion techniques to “form factors”

- Form Factors give probabilities of particle interactions in terms of the final momenta.

- Form factors are similar to amplitudes, but involve an operator that is not on-shell.

*On-shell are momenta that obey $E^2 - p^2 c^2 = m^2 c^4$*

- For gluons $m = 0$. 

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Simplest Example

Operator that gives $g_+g_+$ state.

(A. J. Larkoski and M. E. Peskin, Phys. Rev. D 81, 054010 (2010).)

- For this operator we have

\[
A(O \to g_1+g_2+g_3+) = \frac{(p_1 + p_2 + p_3)^4}{\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle}
\]

- With BCFW only one diagram contributes

- Extend to $n$ particles:

\[
A(O \to g_1+g_2+g_3+\ldots+g_{n+}) = (-1)^{n-1} \frac{(p_1 + p_2 + p_3 + p_4 + \ldots+p_n)^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \ldots \langle n1 \rangle}
\]
Conclusion and Future Goals

- BCFW Recursion Relations is a modern approach to calculating scattering amplitudes.
- BCFW Method is effective!
- Applied BCFW to Form Factors

Goal:
- Study form factors with different operators and same final states.
- Same Operator and different final states.
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